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Eijffinger, S.C.W.; Schaling, E.; Verhagen, W.H.

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# A THEORY OF INTEREST RATE STEPPING: INFLATION TARGETING IN A DYNAMIC MENU COST MODEL

*Sylvester Eijffinger\*, Eric Schaling\*\* and Willem Verhagen\*\*\*<sup>1</sup>*

\*CentER for Economic Research, Tilburg University, College of Europe and Humboldt University of Berlin

\*\* Department of Economics, RAU, PO Box 524, 2006 Auckland Park, Johannesburg, Republic of South Africa

\*\*\* CentER for Economic Research, Tilburg University, PO Box 90153, 5000 LE Tilburg, The Netherlands

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## **Abstract:**

A stylised fact of monetary policy making is that central banks do not immediately respond to new information but rather seem to prefer to wait until sufficient ‘evidence’ to warrant a change has accumulated. However, theoretical models of inflation targeting imply that an optimising central bank should *continuously* respond to shocks. This paper attempts to explain this stylised fact by introducing a small menu cost which is incurred every time the central bank changes the interest rate. It is shown that this produces a relatively large range of inaction because this cost will induce the central bank to take the option value of the status quo into account. In other words, because action is costly the central bank will have an incentive to wait and see whether or not the economy will move closer to the inflation target of its own accord. Next, the paper analyses the implications for the time series properties of interest rates. In particular, we examine the effect of the interest rate sensitivity of aggregate demand, the slope of the Lucas supply function and the variance of demand shocks on the size of the interest rate step and the expected length of the time period till the next interest rate step. Finally, we analyse the effect of menu costs on inflationary expectations. In this respect we find that the economy will suffer from an inflationary bias if the cost of raising the interest rate exceeds the cost of lowering it.

**Keywords:** inflation targeting, dynamic menu costs, uncertainty

JEL Codes: E52, E58

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<sup>1</sup> Correspondence to: Willem Verhagen, phone +31 13 466 8205, fax: +31 13 466 3042, e-mail: [W.H.Verhagen@kub.nl](mailto:W.H.Verhagen@kub.nl) The authors gratefully acknowledge useful discussions with Bouke Buitenkamp and Tony Yates. However, any errors remain our own responsibility.

## 1. Introduction

*“...In sum, given that inflation was forecast to be close to the target in two year’s time and that the outlook beyond then was highly uncertain, the Committee could sensibly wait to gather more information before concluding that policy needed to be changed...”*

Minutes of Monetary Policy Committee Meeting, 5 and 6 August 1998

As a result of the disappointment with monetary targeting and/or fixed exchange rates, many countries have now adopted a regime of (direct) inflation targeting. The use of explicit inflation targets derives its theoretical rationale from the fact that they can overcome credibility problems since they can replicate the results of optimal performance incentive contracts (see Walsh (1995) and Svensson (1997a)). From a theoretical perspective this has also stimulated the research on monetary policy rules which deal with the question how these explicit inflation targets should be translated into monetary policy instruments (see e.g. Taylor (1993,1998), Svensson (1997b) and Haldane (1997)). This literature explicitly recognises the fact that, because of lags in the transmission mechanism, the actual future rate of inflation will not be under direct control of the central bank. Rather, central banks will use their ability to manipulate the (short-term) interest rate to target the expected future inflation rate conditional on all information that is currently available. Consequently, these models also prescribe the appropriate response to a shock to one of the determinants of inflation. In particular, on the assumption that the

central bank only cares about inflation stabilisation it should assess the impact of the shock on the conditional inflation forecast and subsequently change the interest rate so as maintain the equality between the conditional inflation forecast and the assigned inflation target.<sup>2</sup> As a result, the optimal conduct of monetary policy implies that the short-term interest rate will inherit the time-series properties of the determinants of inflation.

However, a stylised fact of actual monetary policy making is that central banks do not immediately change the interest rate in response to new information about the state of the economy.<sup>3</sup> Rather, the instrument of monetary policy tends to remain constant in the face of a changing environment and tends to be changed by discrete amounts while the variables which appear in the central bank's reaction function (e.g. inflation and output) change continuously. Following Bhundia and Yates (1997) we will refer to this phenomenon as *interest rate stepping*. It should be emphasised that this is not the same as interest rate *smoothing*. The latter can be defined as the well-established practice of implementing a desired change in the monetary policy stance in a series of small steps in the same direction rather than taking one single large step all at once.

The purpose of this paper is to reconcile interest rate stepping with optimising behaviour on the part of the central bank and to explore the economic implications of the resulting discrete interest rate changes in a continuously changing environment. To this end we introduce a small 'menu' cost which is incurred every time the central bank changes the

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<sup>2</sup> For a formal treatment of this point see Svensson (1997c).

<sup>3</sup> For useful surveys of this phenomenon see Rudebusch (1995), Goodhart (1996) and Bhundia and Yates (1997).

interest rate. Following the literature on the impact of such costs of decision making on the behaviour of monopolistic price setters (see e.g. Mankiw (1985) and Akerlof and Yellen (1985)), under these conditions it is no longer optimal for the central bank to respond to small deviations from the optimum. Moreover, in a dynamic setting these costs will *induce* the central bank to take the option value of the status quo into account. Obviously, this option value will be irrelevant if action can be taken at no cost since in that case there is nothing to prevent the central bank from keeping inflation equal to the assigned target continuously. Since the cost, once incurred, will not be reversed by an interest change in the opposite direction, there is an incentive for the central bank to wait and see whether or not the economy will move inflation back towards the target of its own accord. As a result, the central bank will allow the inflation rate to fluctuate freely within a certain range.

The paper proceeds as follows, Section 2 outlines a simple closed economy and provides a number of reasons for the existence of menu costs. In Section 3 we present the solution to the model under three different scenarios; a benchmark case where menu costs are absent, the case where the central bank solves a string of unrelated ‘period’ problems and finally the case where the central bank explicitly recognises the intertemporal aspect of its problem. Subsequently, we examine the factors which influence the width of the inflation band. Section 4 examines the implications for the dynamics of short-term interest rates in the light of the empirical literature on this subject. Section 5 solves for the expected rate of inflation and assesses under which conditions the economy will suffer from an inflationary bias. Finally, Section 6 concludes.

## 2. A Simple Closed Economy Model

Consider the following economy in continuous time. Aggregate supply ( $y_t^s$ ) is given by the familiar Lucas-supply function .

$$y_t^s = \beta(\pi_t - \pi^e) \tag{1}$$

In this equation the natural rate of output ( $y^*$ ) has been normalised to zero. The parameter  $\beta$  measures the slope of the Lucas supply function,  $\pi_t$  is the (instantaneous) rate of inflation rate and  $\pi^e$  denotes inflationary expectations. As indicated by the absence of a time subscript inflationary expectations do not depend on any particular point in time. One can think of this as the result of the existence of fixed nominal wage contracts. More precisely, agents will determine the expected rate of inflation using the long run probability density function of inflation conditional on the central bank's optimal monetary policy.<sup>4</sup> The exact factors which determine  $\pi^e$  will be discussed in Section 5. For now we note that the central bank will take inflationary expectations as given when setting the interest rate. Aggregate demand ( $y_t^d$ ) is modelled as follows:

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<sup>4</sup> Formally, let  $F$  be the information set available to private agents containing the information they have about optimal monetary policy and let  $g(\pi|F)$  be the long run probability density function of inflation conditional on this information set. Then we have:

$$\pi^e = E(\pi | F) = \int_{-\infty}^{\infty} \pi g(\pi | F) d\pi$$

$$y_t^d = -\alpha(i_t - \pi_t) + \eta_t \quad (2)$$

Here  $i_t$  is the instrument of the central bank, i.e. the nominal interest rate which expresses the monetary policy stance (e.g. the UK base rate, the US Federal Funds Target or the ECB's repo rate). The parameter  $\alpha$  measures the sensitivity of aggregate demand to the ex post real interest rate and  $\eta_t$  is an exogenous demand shock which follows a driftless Brownian motion:

$$d\eta = \sigma dw \quad (3)$$

There is no particular economic reason for assuming a continuous time random walk on the demand shock. However, unlike more sophisticated processes (e.g. exhibiting mean-reversion) this assumption will allow us to compute a relatively simple analytic solution to the central bank's problem.

As far as the preferences of the central bank are concerned, it is assumed that there is a basic trade-off between deviations of the rate of inflation from the assigned target ( $\pi^*$ ), on the one hand, and costs which are incurred whenever the interest rate is *changed*, on the other. In view of this trade-off the central bank will minimise the following intertemporal loss function:

$$L(\pi) = E \left\{ \int_0^\infty e^{-\delta t} (\pi_t - \pi^*)^2 dt + \sum_j C e^{-\delta t_j} | \pi_0 = \pi \right\} \quad (4)$$

Here  $\delta$  is the central bank's discount rate (which is inversely related to the policy horizon) and  $t_j$  denotes the instants where the central bank decides to change in the interest rate. Each time this happens the central bank will incur a cost which is equal to 'C' for which it holds that C is small (i.e.  $C \sim h$ ). Apart from these costs, the central bank is assumed to engage in strict inflation targeting.<sup>5</sup> While this may seem a restrictive assumption since virtually every central bank also cares about output fluctuations (at least around the natural rate of output) it should be emphasised that in our model, which features only demand shocks, inflation stabilisation *implies* output stabilisation.

The presence of a small cost of changing the interest rate in the central bank's loss function can be rationalised on a number of grounds. First of all, the central bank could partly internalise the costs incurred by agents who are bound into fixed nominal interest rate contracts. For instance, Cukierman (1990, pp. 113) argues that the central bank will be "... *concerned with the predictability of interest rates rather than with their level.*". The reason for this resides in the traditional task of the banking system to provide liquidity by transforming short-term liabilities into long-term assets. This implies that the interest rates charged on the asset side of the balance sheet are fixed for relatively long periods while the interest rates paid on the liability side are likely to change every time the official interest rate changes. Stable official interest rates will therefore reduce the probability of an interest rate mismatch .

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<sup>5</sup> In reality, the actual future rate of inflation will of course never be under perfect control by the central bank. However, Svensson (1997) has shown that inflation targeting implies that the conditional forecast of inflation becomes the intermediate target of monetary policy. The latter *can* of course be perfectly and instantaneously controlled by the central bank.



Secondly, as argued by Crockett (1994) central bankers may also face a ‘psychological’ cost when they change their minds, for instance since this makes them vulnerable to allegations of inconsistency or incompetence. As argued by Goodhart (1999) this cost is likely to be prohibitive when the need for a change in the monetary policy stance is not very obvious to outside observers (i.e. when inflation or the inflation forecast is close to the target and output is close to potential). In that case, given the random walk nature of news about these variables, there is a considerable chance that an interest change that is optimal today will have to be reversed in the near future. This might give the impression that the central bank is uncertain about the appropriate direction for monetary policy. Moreover, despite a considerable degree of *formal* independence, the central bank may still be under pressure from politicians not to raise interest rates. As a consequence, the central bank will also be reluctant to lower interest rates because once they are lowered it may be ‘politically difficult’ to increase them again.<sup>6</sup>

Finally, there is an argument related to the way the interbank money market works. The Fed, for instance, announces a target for the Fed Funds Rate. Unpredictable shifts in the demand curve for central bank balances will cause the Fed Funds Rate to fluctuate randomly around this target (this is because the Fed subsequently corrects these shifts through open market operations to maintain the Fed Funds Rate equal to the target on average). If the Fed were to react optimally to every bit of economic news that comes in it would have to change the Fed Funds Target frequently by probably only a few

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<sup>6</sup> According to Huizinga and Eijffinger (1999) there is also a strategic argument for not changing the monetary policy stance in response to every (supply) shock since this will lower inflationary expectations.

basispoints. Given the afore-mentioned volatility of the actual Fed Funds Rate this would reduce the information value of interest rate changes which presents an incentive to the Fed to economise on the number of steps to be taken.

### 3. Solution under Static and Rational Expectations

#### 3.1 no menu costs

As a benchmark we will first solve for the equilibrium in the absence of menu costs ( $C = 0$ ). From equations (1) and (2) we can derive the following reduced form for inflation:

$$\pi_t = \frac{\beta}{\beta - \alpha} \pi^e - \frac{\alpha}{\beta - \alpha} i_t + \frac{1}{\beta - \alpha} \eta_t \quad (5)$$

In order to rule out a perverse response of inflation to its determinants we need to assume that  $\beta > \alpha$ . Obviously, the central bank's intertemporal loss function (4) will be minimised if it sets  $i_t$  so as to ensure that the condition  $\pi_t = \pi^*$  holds continuously.<sup>7</sup> Substituting this condition in equation (5) and solving for  $i_t$  yields the following endogenous instrument rule

$$i_t = \pi^* + \frac{\beta}{\alpha} (\pi^e - \pi^*) + \frac{1}{\alpha} \eta_t \quad (6)$$

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<sup>7</sup> Following Svensson (1997) this equality is simply the optimal (intermediate) target rule.

This equation is very similar to the Taylor rule (Taylor (1993)) in the sense that it expresses the optimal value of the central bank's instrument as a linear function of the determinants of inflation. In particular, the interest rate will inherit the time-series properties of the demand shock and will therefore also follow a driftless Brownian motion.<sup>8</sup> It appears that the afore-mentioned stability condition concerning the ratio of the slope of the Lucas supply function and the interest rate sensitivity of aggregate demand ( $\beta/\alpha$ ) implies that the response-coefficient for  $(\pi^e - \pi^*)$  will be strictly greater than one. This is a well-known and robust condition for stability in the literature on monetary policy rules (see Taylor (1998)).

Plugging the optimal rule (6) back into the reduced form for inflation (5) yields:  $\pi_t = \pi^*$ . Since wage setters know that the central bank will always keep inflation equal to the target they will determine the expected rate of inflation as follows:  $\pi^e = E(\pi) = \pi^*$ . As a result, the economy will permanently be at the equilibrium where it holds that  $\pi_t = \pi^e = \pi^*$  and  $y = y^* = 0$ .

### 3.2 positive menu costs

If changing the interest rate is costly, it will no longer be optimal to do so if the deviation of the inflation rate from the target is small (in a manner to be made more precise later). In other words, there will be a trade-off between losses arising from deviations of

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<sup>8</sup> As we will show later, even in the presence of menu costs this rule describes the long-run behaviour of the interest rate. Since the demand shock follows a continuous time random walk and since inflation will be stationary as a result of optimal monetary policy it follows that both the nominal and the ex post real interest rate will be non-stationary.

inflation from its target, on the one hand, and losses stemming from interest rate adjustments, on the other. As a starting point for the analysis we will compute the solution for the inflation rate under the condition that the interest rate is kept constant. Immediately after a change in the interest rate (at, say,  $t=0$ ) the economy will be in a situation where the inflation rate is equal to the target ( $\pi_0=\pi^*$ ). Without loss of generality we normalise the initial value of the demand shock to zero ( $\eta_0 = 0$ ). Inflationary expectations are fixed and equal to  $\pi^e$ . Plugging these parameter values into the optimal instrument rule (6) yields the following for the nominal interest rate at  $t=0$ :

$$i_0 = \pi^* + \frac{\beta}{\alpha}(\pi^e - \pi^*) \quad (7)$$

Substituting this expression into the reduced form equation for inflation (5) we obtain an expression for  $\pi_t$  which holds as long as the interest rate is maintained at the value specified in equation (7). Since we can repeat this procedure for *every* instant the interest rate is changed we can derive the following general expression for the rate of inflation which holds for all periods between interest rate changes:

$$\begin{aligned} \pi_t - \pi^* \equiv x_t = \theta \varepsilon_t \quad ; \quad \theta \equiv \frac{1}{(\beta - \alpha)} \\ d\varepsilon = \sigma dw \quad ; \quad \varepsilon_0 = \varepsilon_{t_j} = \eta_0 = 0 \end{aligned} \quad (8)$$

Here  $\varepsilon_t$  is defined as the stochastic shock to the *inflation gap* ( $x_t$ ). This shock can be

thought of as a re-normalised value of the demand shock ( $\eta_t$ ). Starting from  $t=0$  the shock to the inflation gap will be equal to the demand shock (i.e.  $\varepsilon_t = \eta_t$  for  $i_t = i_0$ ). Now suppose that at  $t=\tau$  the central bank decides to change the interest rate. Obviously, the new interest rate will be set so as to bring inflation back to the target (i.e. it will hold that  $i_\tau = \pi^* + (\beta/\alpha) (\pi^e - \pi^*) + \eta_\tau/\alpha$ ). Since at the time of resetting we need to have  $\varepsilon_\tau=0$  in equation (8), it will hold that:  $\varepsilon_t = \eta_t - \eta_\tau$  for  $i_t = i_\tau$ . Of course, this normalisation of the demand shock can be applied to *all* instants in which the interest rate is changed (i.e. for all  $t_j$ ).

Now suppose the central bank ignores the fact that it is dealing with a *dynamic* optimisation problem and simply solves a string of unrelated ‘period’ optimisation problems instead. In other words, the central bank will treat  $\varepsilon_t$  as a ‘once and-for-all shock’ or, equivalently, it has static expectations in the sense that it does not take the stochastic properties of  $\varepsilon_t$  into account. At each point in time, the central bank will then compare the discounted present value of the flow cost ( $x_t^2/\delta$ ) to the cost of changing the interest rate (C). Hence, under static expectations, the central bank will set  $i_t$  according to the optimal rule (6) if the following condition is met:

$$\frac{\theta^2 \varepsilon_t^2}{\delta} > C \quad \Leftrightarrow \quad (\theta \varepsilon_t)^2 = (\pi_t - \pi^*)^2 > s = C\delta \quad (10)$$

Consequently, even under static expectations ‘menu’ costs which are of second-order smallness ( $C \sim h^2$ ) will lead to a range of inaction which is of first-order smallness ( $s \sim h$ ).

As noted by Dixit (1991) it is in this sense that small menu costs produce relatively large effects.

Under rational expectations the central bank will recognise the intertemporal aspect of its problem and will explicitly take the stochastic process driving the demand shock into account. In other words, if the loss stemming from the inflation gap passes the ‘static expectations threshold’ in equation (10) it is no longer optimal to change the interest rate and incur the cost of doing so. This is because the central bank has the *option* to wait and see whether or not the economy will move inflation back to the target level of its own accord. Similar to the case where the central bank has ‘static’ expectations, the optimisation problem boils down to choosing a threshold level for the inflation gap (b) which will trigger a change in the interest rate. On the assumption that the cost of raising the interest rate is equal to the cost of lowering it, the upper and lower threshold levels imply a symmetric band within which inflation is allowed to move according to the process defined in equation (8). Moreover, because the cost of changing the interest rate does not depend on the magnitude of the change (i.e. these costs stem simply from the fact that there *is* a change in the interest rate) the inflation gap will be set to zero whenever it hits one of the thresholds.

First of all, to solve the central bank’s problem (4) we now have to translate the stochastic properties of the shock to the inflation gap ( $\epsilon_t$ ) into stochastic properties for the inflation gap itself ( $x_t$ ). Applying the rules of stochastic calculus to equation (8) we can write:<sup>9</sup>

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<sup>9</sup> When  $x = g(\epsilon)$ , where  $\epsilon$  follows a driftless Brownian (see equation (3)), it will hold that:

$$dx = \theta \sigma dw \quad (11)$$

Next, we would like to find an expression for the loss function (4) which can be minimised with respect to the central bank's choice variable (b). We realise that the interest rate will not be changed as long as the inflation gap is strictly within the band. Hence, for any  $x \in (-b, b)$  we can express the RHS of equation (4) by means of the Bellman equation:

$$L(x) = x^2 dt + e^{-\delta dt} E\{L(x + dx)\} \quad (12)$$

Expanding the RHS of this equation and using Ito's lemma (see Appendix A) yields a second-order differential equation:

$$\frac{1}{2} \theta^2 \sigma^2 L''(x) - \delta L(x) + x^2 = 0 \quad (13)$$

In Appendix A it is shown that the general solution to this equation can be expressed as follows:

$$L(x) = \frac{x^2}{\delta} + \frac{\theta^2 \sigma^2}{\delta^2} + A(e^{\gamma x} + e^{-\gamma x}) \quad ; \quad \gamma \equiv \frac{\sqrt{2\delta}}{\theta \sigma} \quad ; \quad A = A(b) \quad (14)$$

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$$dx = \left[ \frac{1}{2} g''(\varepsilon) \sigma^2 \right] dt + g'(\varepsilon) \sigma dw$$

The first two parts on the RHS denote the expected present value of the loss function under the condition that the interest rate is never changed. Consequently, the third term on the RHS captures the value of being able to make interest rate adjustments. In particular, the effect of the threshold level  $b$  on the intertemporal loss function  $L(x)$  will be fully incorporated in the constant of integration ( $A$ ).

It now remains to solve for the constant of integration  $A$  and the threshold level  $b$  simultaneously. Following Dixit (1991,1993) there are two conditions which pin down these parameters. First of all, the Value Matching Condition (VMC) which says that in the optimum the reduction in the value of  $L(x)$  obtained by exercising control should equal the cost of changing the interest rate. In other words, the optimal choice of the threshold level implies that there are no discontinuities in the intertemporal loss function (if there were ‘discrete jumps’ in  $L(x)$  for a particular choice of  $b$  this choice would obviously not be optimal). Applying this to equation (14) we obtain:

$$L(b) - L(0) = C \quad \Leftrightarrow \quad A(e^{\gamma b} + e^{-\gamma b} - 2) = C - \frac{b^2}{\delta} \quad (15)$$

Secondly, the Smooth Pasting Condition (SPC) which requires the graphs of the  $L(x)$  and  $C$ -functions to meet tangentially at the point where  $x=b$ . This can be understood by observing that, for expression (14) to be minimised, we need the first order condition  $A'(b)=0$ . Differentiating the Value Matching condition with respect to  $b$  and using this



first order condition yields:<sup>10</sup>

$$L'(b) = 0 \quad \Leftrightarrow \quad \gamma A (e^{\gamma b} - e^{-\gamma b}) = -\frac{2b}{\delta} \quad (16)$$

Since both equation (15) and (16) are highly non-linear, A and b can generally only be solved numerically. However Dixit (1991) has shown that the solution for b can be approximated analytically (see Appendix A), this yields:

$$b = \left( \frac{6C\sigma^2}{(\beta - \alpha)^2} \right)^{\frac{1}{4}} \quad (17)$$

Hence, under rational expectations fourth-order menu costs ( $C \sim h^4$ ) will have a first-order effect on the band of inaction ( $b \sim h$ ). The reason is that under rational expectations the policymaker will take the *option value of the status quo* into account. In particular, when the inflation gap hits the ‘static expectations threshold’ specified in equation (10) it is no longer optimal for the central bank to reset the inflation gap back to zero by incurring the small cost equal to C. Instead at this point the central banker will wait for a small amount of time (dt) during which he will receive new information about the state of the economy. More precisely, the central bank will be able to see if the inflation gap moves back

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<sup>10</sup> Note that this condition proves that barriers will reduce the value of the loss function (relative to the value obtained in the situation where control is never exercised) since equation (16) will only hold for  $A < 0$ .

towards zero of its own accord. Consequently, there will be a trade-off between the 'period' flow cost stemming from the inflation gap, on the one hand, and the cost of exercising control *plus* the option value of the status quo, on the other.

This is illustrated in Figure 1 which depicts the situation immediately after an interest rate step has been taken. The aggregate supply curve ( $y^s$ ) is drawn for the situation where  $\pi^e = \pi^*$ . The demand shock has the effect of shifting the aggregate demand curve ( $y^d$ ) randomly along the aggregate supply curve. If there are no costs to changing the interest rate ( $C=0$ ) the central bank will offset each shock so as to preserve the situation where inflation is equal to the target. However, if changing the interest rate is costly, the demand curve will be allowed to shift around until the rate of inflation hits one of the thresholds.

[insert Figure 1]

Equation (17) allows us to examine the effect of structural and preference parameters on the threshold level for the inflation gap:

**PROPOSITION 1:**

The inflation gap threshold ( $b$ ) will increase if:

1. The cost of changing the interest rate ( $C$ ) increases
2. The volatility of the demand shock ( $\sigma$ ) increases
3. The slope of the Lucas supply function ( $\beta$ ) decreases
4. The interest rate sensitivity of aggregate demand ( $\alpha$ ) increases

The proof of this proposition follows immediately from equation (17). Obviously, an increase in the cost of changing the interest rate will induce the central bank to accept a larger inflation gap before taking action. Next, the effect of the volatility of the demand shock, the slope of the Lucas supply function and the interest rate sensitivity of aggregate demand can be understood from the way they affect the volatility of the stochastic process driving the inflation gap as described in equation (11). This is because an increase in the volatility of the inflation gap will also increase the option value of the status quo.

First of all, since the inflation gap is driven by the demand shock, an increase in the volatility of the demand shock ( $\sigma$ ) will spill over into higher inflation gap volatility. Next, a decrease in the slope of the Lucas supply function ( $\beta$ ) will enhance the effect of a given demand shock on inflation since now a larger part of this shock will be absorbed by inflation at the expense of the effect on output. Finally, if aggregate demand becomes more sensitive to the ex post real interest rate ( $i_t - \pi_t$ ) this will enhance the well-known ‘vicious circle of instability’ by which an increase in inflation will increase aggregate demand through the erosion of real interest rates thereby fuelling a further increase in inflation.

#### 4. Implications for the Dynamics of Short-Term Interest Rates

The behaviour of the central bank's key interest rate and the implication of this behaviour for longer-term interest rates has been extensively studied in the empirical literature.<sup>11</sup> For instance, Rudebusch (1995) provides a survey of empirical tests of the expectations hypothesis of the term structure of interest rates, the upshot of which is that term spread predicts future movements in interest rates fairly well in the very short-run (up to 1 month) and in the long run (2 years and longer). The first finding can be attributed to the tendency of many central banks to smooth interest rates (i.e. to implement the required increase or decrease in a series of small steps rather than all at once). The second observation can be explained by the fact that in the long run the level of interest rates will be determined by the central bank's desire to achieve its ultimate monetary policy goals. Since the latter are to a considerable extent known to the public, agents will be able to predict interest rate movements over long horizons with a reasonable degree of accuracy.

However, in the medium run the predictive ability of the term spread is very poor which led many researchers to reject the rational expectations theory of the term structure. Mankiw and Miron (1986) have argued that the lack of predictive ability can be explained by explicitly taking the manner in which the central bank controls interest rates into account. In particular, they suggest the Fed imparts random walk behaviour to the Federal Funds Target in which case the hypothesis of rational expectations implies

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<sup>11</sup> Recent examples are Rudebusch (1995), Balduzzi, Berola and Foresi (1997) and Balduzzi, Bertola, Foresi and Klapper (1998).

precisely that future short-term interest rates should not be predictable. This idea has been extended by Rudebusch (1995), Balduzzi et al (1997) and Balduzzi et al (1998). These authors explicitly model the process generating the central bank's target interest rate by postulating that on any given day within the sample period, there will a relatively small but equal probability of a target change of fixed size in either direction.<sup>12</sup> Moreover, Balduzzi et al (1998) document a new stylised fact, namely that the volatility and persistence of the spread increases with the maturity of the loan. They show that spreads of longer-term (e.g. 3 or 6 month) rates from the target are mainly driven by expectations of future target changes. When a target change takes place, all 'adjustment pressures' will be released. However, immediately thereafter the market starts to receive new information which leads to partial predictability of the next target change. Obviously, in view of the fact that the central bank engages in interest rate stepping, the impact of this information on the spread will increase with the maturity of the debt instrument.

In view of this description of interest rate stepping in the empirical literature it is interesting to investigate the factors which determine the size of the interest rate step, the expected duration till the next target change and the extent to which the next target change is predictable. First of all, in our model interest rate steps will always be of a given and fixed size.<sup>13</sup> This is because the interest rate will be reset at a new optimal level if and only if the inflation gap hits one of the thresholds (i.e. if it holds that  $|x_t| = b$ ).

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<sup>12</sup> This formulation abstracts from interest rate smoothing considerations since these will induce a relatively high probability of a target change in the same direction during the first month after a target change (see Rudebusch (1995)).

<sup>13</sup> Rudebusch (1995) shows that in reality the size of the interest rate step is drawn from a discrete

Suppose that starting from  $t=0$ , the inflation gap first hits one of the barriers at  $t = \tau$ .

From equation (8) it can be seen that this implies that  $|\varepsilon_\tau| = b(\beta - \alpha)$ . Plugging this expression into the optimal interest rate rule (6) yields:

$|i_\tau| = \pi^* + (\beta/\alpha)(\pi^e - \pi^*) + b(\beta - \alpha)/\alpha$ . Subtracting the expression for  $i_0$  obtained in equation (7) and using the expression for  $b$  in equation (17) will yield the following expression for the absolute value of the interest rate step:

$$|i_\tau - i_0| = \frac{(6C\sigma^2)^{\frac{1}{4}}}{\alpha(\beta - \alpha)^{\frac{1}{2}}} \quad (18)$$

### PROPOSITION 2:

The absolute value of the interest rate step ( $|i_\tau - i_0|$ ) will increase if:

1. The cost of changing the interest rate ( $C$ ) increases
2. The volatility of the demand shock ( $\sigma$ ) increases
3. The slope of the Lucas supply function ( $\beta$ ) decreases
4. The interest rate sensitivity of aggregate demand ( $\alpha$ ) increases for  $\beta/\alpha < 3/2$  or decreases for  $\beta/\alpha > 3/2$

*Proof:* see Appendix D.

The intuition is that an increase in  $C$ , an increase in  $\sigma$  or a decrease in  $\beta$  will induce an increase in the threshold level ( $b$ ). Hence, a *larger* interest rate step will be needed when the inflation gap hits one of the barriers. As far as an increase in  $\alpha$  is concerned there are two opposing effects. On the one hand this will cause the threshold level ( $b$ ) to go up. On the other hand, since aggregate demand will be more sensitive to interest rate changes, a smaller step will be needed for any *given* value of the threshold which will tend to decrease the size of the interest rate step. The model predicts that the first effect will dominate if the reaction coefficient for  $(\pi^e - \pi^*)$  in the optimal interest rate rule (6) is ‘relatively low’ (i.e. for  $1 < \beta/\alpha < 3/2$ ).

Next, we can investigate the factors which affect the expected period of time that will elapse before the next interest rate step is taken ( $T(x)$ ). In Appendix C it is shown that for symmetric threshold levels ( $-b, b$ ) this is given by:

$$T(x) = \frac{(\beta - \alpha)\sqrt{6C}}{\sigma} - \frac{(\beta - \alpha)^2 x^2}{\sigma^2} \quad (19)$$

The following proposition summarises the effect of several model parameters on  $T(x)$ :

**PROPOSITION 3:**

The expected time period that will elapse before the next interest rate step is taken ( $T(x)$ ) will increase if:

1. the cost of changing the interest rate ( $C$ ) increases
2. the slope of the Lucas supply function ( $\beta$ ) increases for  $|x| < b/\sqrt{2}$  or decreases for  $|x| > b/\sqrt{2}$
3. the interest rate sensitivity of aggregate demand ( $\alpha$ ) decreases for  $|x| < b/\sqrt{2}$  or increases for  $|x| > b/\sqrt{2}$
4. the volatility of the demand shock ( $\sigma$ ) decreases for  $|x| < b/\sqrt{2}$  or increases for  $|x| > b/\sqrt{2}$

*Proof:* see Appendix D

An increase in the costs of control ( $C$ ) will increase the threshold level ( $b$ ) because of which it will take longer before the inflation gap reaches one of the threshold levels. The result for the parameters  $\beta, \alpha$  and  $\sigma$  is basically the outcome of two opposing forces. On the one hand, an increase in  $\beta$ , a decrease in  $\alpha$  and/or a decrease in  $\sigma$  will reduce the volatility of the inflation gap (see equation (11)). This will increase the expected time period that will elapse before the interest rate is reset for any *given* value of the threshold level ( $b$ ). However, there is also an indirect effect since a decline in the volatility of the



stochastic process driving the inflation gap will reduce the threshold level itself (see Proposition 1). All else equal, this will reduce the average time till the next interest rate step.

Which one of these two effects dominates depends on the current value of the inflation gap ( $x$ ). The model predicts that a decrease in volatility will increase the expected duration of the current monetary policy stance if the inflation gap is relatively small (i.e. if  $|x| < b/\sqrt{2}$ ). In particular, this will hold for the *average duration between two consecutive interest rate steps* ( $T(0)$ ) which is equal to the first term on the RHS of equation (19). In the empirical literature the probability of a target change during any given day in the sample period is usually estimated using the empirical frequency of target changes (i.e. the number of target changes divided by the number of business days in the sample, see e.g. Balduzzi et al (1997)). Consequently, our model identifies some of the factors that determine this probability since the latter will be inversely related to  $T(0)$ .

#### **COROLLARY 1:**

The average duration between consecutive interest rate steps ( $T(0)$ ) will be *increasing* in the slope of the Lucas supply function ( $\beta$ ) and the cost of changing the interest rate ( $C$ ) and *decreasing* in the interest rate sensitivity of aggregate demand ( $\alpha$ ) and the volatility of the demand shock ( $\sigma$ ).

Finally, to obtain an indication of the predictability of the next interest rate step we can compute the probability that the interest rate will be lowered next time  $Q(x)$ . Suppose that in general the cost of raising the interest rate ( $C_h$ ) differs from the cost of lowering it ( $C_l$ ).<sup>14</sup> This will lead to an optimal range of inaction in the interval  $(-a, b)$  where  $a, b > 0$ . An asymmetry in the cost technology may arise because of the interaction between the desires of politicians and the central bank. For instance, when the latter is to some extent politically subservient, the cost of raising the interest rate may very well exceed the cost of lowering it. Raising the interest rate is politically unpopular while lowering it may yield electoral benefits. The reverse situation may arise when the central bank wants to assert its independence in the face of politicians clamouring for interest rate cuts. In Appendix C it is shown that for  $x \in (-a, b)$ , the probability that the interest rate will be decreased when it is reset ( $Q(x)$ ) is given by:<sup>15</sup>

$$Q(x) = \frac{b-x}{b+a} \quad (20)$$

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14 This would imply:  $L(\pi) = E \left\{ \int_0^\infty e^{-\delta t} (\pi_t - \pi^*)^2 dt + \sum_k C_h e^{-\delta t_k} + \sum_m C_l e^{-\delta t_m} \mid \pi_0 = \pi \right\}$

where  $t_k$  denotes the instants where the interest rate is raised and the central bank incurs a cost equal to  $C_h$  while  $t_m$  denotes the instants in which it is lowered yielding a cost equal to  $C_l$ . In that case we have two Value Matching Conditions ( $L(-a) - L(0) = C_l$  and  $(L(b) - L(0) = C_h)$ ) and two Smooth Pasting Conditions ( $L'(-a) = L'(b) = 0$ ) to determine the two barriers  $a$  and  $b$  and the two constants of integration. It can easily be shown that  $b'(C_h) > 0$  and that  $a'(C_l) > 0$  and therefore  $C_h > C_l$  implies  $b > a$ .

<sup>15</sup> Of course the probability of an interest rate increase at the next step is simply the complementary

First of all, from this equation it can be seen that interest rate changes are perfectly anticipated by the time they occur (i.e.  $Q(-a) = 1$  and  $Q(b)=0$ ). This is because in our model the central bank does not have an information advantage over the public. In particular, this means that there is no uncertainty on the part of private agents concerning the position of the thresholds which allows them to anticipate interest rate changes with certainty the instant before they are implemented. Next, the effect of the cost of raising and the cost of lowering the interest rate ( $C_h$  and  $C_l$  respectively) on the probability of an interest rate decrease at the next step is summarised by the following proposition:

**PROPOSITION 4:**

The probability of an interest rate decrease at the next step ( $Q(x)$ ) will increase if the cost of lowering the interest rate ( $C_l$ ) decreases and/or if the cost of raising the interest rate ( $C_h$ ) increases.

*Proof:* see Appendix D.

The intuition is that the absolute value of the upper threshold will exceed the absolute value of the lower threshold if the cost of raising the interest rate exceeds the cost of lowering it. This means that the probability that the inflation gap will first reach the lower threshold will increase for any given rate of inflation.

## 5. The Effect of Dynamic Menu Costs on Inflationary Expectations

Since the expected rate of inflation is locked into nominal wage contracts it will not respond to short-run fluctuations in aggregate demand and/or any one *particular* interest rate response to these fluctuations. In other words, the expected rate of inflation will be determined by agents' beliefs concerning the long-run characteristics of monetary policy. In particular, they know the preferences of the central bank from which they can deduce the range of inaction and, consequently, the long run probability density function for inflation conditional on the thresholds chosen by the central bank. This, in turn, allows them to compute a rational expectation of inflation.

In Appendix C it is shown that for thresholds  $-a$  and  $b$ , the long run probability density function for the inflation gap  $\phi(x)$  will be as follows:

$$\phi(x) = \begin{cases} \frac{2(a+x)}{a(a+b)} & \text{for } -a \leq x < 0 \\ \frac{2}{(a+b)} & \text{for } x = 0 \\ \frac{2(b-x)}{b(a+b)} & \text{for } 0 < x \leq b \end{cases} \quad (20)$$

This probability density function is depicted in Figure 2.

[insert Figure 2]

Using this we can compute  $\pi^e = E(x) + \pi^*$ , where the expected value of the inflation gap ( $E(x)$ ) will be equal to:

$$E(x) = \int_{-a}^b x\phi(x)dx = \frac{b^2 - a^2}{3(a+b)} \quad (21)$$

From this we can infer the following relationship between the inflationary bias and the costs of raising or lowering the interest rate:

**PROPOSITION 5:**

The economy will suffer from an upward (downward) inflationary bias ( $\pi^e > < \pi^*$ ) if the cost of raising the interest rate exceeds (is smaller than) the cost of lowering it ( $C_h > < C_l$ ).

*Proof:* see Appendix D

In most models an inflationary bias arises because the policymaker faces a systematic temptation to create surprise inflation once nominal contracts are signed. This is because unanticipated inflation enables the policy maker to pursue various real objectives<sup>16</sup> (e.g. an output level which is higher than the natural output level). In this model the central bank is *not* tempted to cheat the public since its only *ultimate* monetary policy goal is to stabilise inflation. The introduction of a small menu cost does not alter this basic fact, even though it means that control will no longer be exercised continuously. In that case, provided the cost structure is symmetric (implying  $a = b$ ) inflation will not deviate

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<sup>16</sup> For a survey see Cukierman (1992, Chs. 2-7)

*systematically* from its target ( $\pi^*$ ) because of which the latter will feature as the expected rate of inflation which is locked into nominal wage contracts. All this implies that observationally the economy will move along a stable Phillips curve of the form  $\pi_t = \pi^* + (1/\beta)y_t$ . This relationship is stable *precisely* because the central bank does not systematically try to take advantage of this relationship<sup>17</sup>.

However, if for reasons mentioned earlier the cost of raising the interest rate is higher than the cost of lowering it, the probability mass to the right of the point where the inflation gap is zero ( $\phi(0)$ ) will exceed the probability mass to the left of this point (this is the situation depicted in Figure 2). Taking this into account wage setters realise that the tendency to maintain the current policy stance longer in the face of upward inflationary pressures will produce an average rate of inflation which is higher than the target. At the risk of repetition it should be noted that this inflationary bias arises even though the central bank does *not* face a systematic temptation to generate surprise inflation. The optimal instrument rule (6) is fully credible (in the sense that the public faces no uncertainty about this rule) and implies that the inflation gap will be set equal to zero every time the central bank decides to ‘switch this rule on’. Moreover, changes in the monetary policy stance (i.e. the interest rate) are always *fully* anticipated the instant before they occur.

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<sup>17</sup> Of course, if the central bank were to try and take advantage of this relationship it would break down as a result of the Lucas critique. In other words, this stable Phillips curve would fall victim to Goodhart’s law that ‘...any statistical regularity will tend to collapse once pressure is placed upon it for control purposes...’ (Goodhart (1989))

## 6. Summary and Conclusion

This paper studies a simple model of inflation targeting in which inflation stabilisation features as the only ultimate goal of monetary policy. In addition, the central bank incurs a small cost every time the monetary policy stance (i.e. the short-term interest rate) is changed. Since this cost will induce the central bank to take the option value of the status quo into account it will have a considerable effect on the inflation outcome. In particular, costs of fourth-order smallness will have a first order effect on the band within which inflation is allowed to fluctuate without a change in the interest rate. This band provides an explanation for the well-documented central bank practice of interest rate stepping. We examine how the width of this band depends on the cost of changing the interest rate and the volatility of the inflation process. The latter will be determined by the volatility of the underlying demand shocks, the slope of the Lucas supply function and the interest rate sensitivity of aggregate demand.

In the empirical literature interest rate stepping has been used extensively to offer a ‘rational expectations consistent’ explanation for the failure of the term spread to predict future movements in short-term interest rates. In view of these results we assessed the factors that determine the size of the interest rate step, the expected time till the next interest rate step and the probability that interest rates will fall next time the central bank decides to take action. Some of the propositions we derive in this respect lend themselves to empirical testing. For instance, the model predicts that the size of the interest rate step will be increasing in the cost of changing the interest rate and the volatility of the demand shock and decreasing in the slope of the Lucas supply curve. Similarly, the average

duration between two consecutive steps will be decreasing in the interest rate sensitivity of aggregate demand and the volatility of the demand shock and increasing in the slope of the Lucas supply function and the cost of changing the interest rate. Finally, we examine the effect of these ‘menu’ costs on inflationary expectations. We show that the economy will suffer from an inflationary bias if the cost of raising the interest rate exceeds the cost of lowering. This result is interesting since it shows that an inflationary bias can arise even if the central bank does not try to create surprise inflation in pursuit of various real objectives.

In line with the literature on monetary policy rules, our model clearly distinguishes between the interest rate as the control variable and the rate of inflation as a state variable. However, it differs from most other models in assuming that inflation is instantaneously and perfectly controllable, i.e. it abstracts from lags in the transmission process. Nevertheless, in these models the conditional inflation forecast, which serves as the intermediate target of monetary policy, *can* be perfectly and instantaneously controlled. Hence, in our view the rate of inflation in our model is best viewed as the conditional inflation forecast when considering the implications of the model for the real world. In this sense the model provides an explanation for the existence of bands for the intermediate target of monetary policy even if this intermediate target itself is perfectly controllable.



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## Appendix A: Derivation of the optimal band of inaction

Using the fact that  $e^{-\delta dt} \approx 1 - \delta dt$  we can rewrite equation (13) as follows:

$$\begin{aligned} L(x) &= x^2 dt + (1 - \delta dt) \{L(x) + E(L(x + dx) - L(x))\} \\ &= L(x) + x^2 dt - \delta L(x) dt + E(dL(x)) - \delta dt E(dL(x)) \end{aligned} \quad (\text{A.1})$$

Since  $dx = \theta \sigma dw$ , by Ito's Lemma it holds that:

$$E(dL(x)) = \frac{1}{2} \theta^2 \sigma^2 L''(x) dt \quad (\text{A.2})$$

Substituting this equation in (A.1), ignoring terms which are small relative to  $dt$  and subsequently dividing by  $dt$  will yield the second-order differential equation (13) in the main text. The solution to this equation consists of the sum of a particular solution ( $L_p(x)$ ) and the general solution of the homogeneous part:

$$L(x) = L_p(x) + Ae^{q_1 x} + Be^{q_2 x} \quad (\text{A.3})$$

Here  $A$  and  $B$  are constants to be determined and  $q_1$  and  $q_2$  are the roots of the characteristic equation of the homogeneous part.

Since the forcing term is quadratic in  $x$ , we try the following particular solution:

$$L_p(x) = d_0 x^2 + d_1 x + d_2 \quad (\text{A.4})$$

Plugging the resulting expressions for  $L''(x)$  and  $L(x)$  in equation (13) and subsequently equating coefficients across equations (13) and (A.4) yields:  $d_0 = 1/\delta$ ,  $d_1 = 0$  and  $d_2 = (\theta^2 \sigma^2)/\delta^2$ . As in Dixit (1993) the resulting particular solution can be thought of as the present value of the intertemporal loss function under the condition that control is never exercised<sup>18</sup>. Consequently, the effect of barriers will be fully captured by the complementary function. To find this function we solve the characteristic equation of the homogeneous part to obtain the following expression for the characteristic roots:

$$\frac{1}{2} \theta^2 \sigma^2 q^2 - \delta = 0 \quad \Leftrightarrow \quad q_{1,2} = \pm \frac{\sqrt{2\delta}}{\sigma\theta} \quad (\text{A.5})$$

Next, regarding equation (A.3) we note that the threshold level (b) will only affect the constants A and B, since the band is symmetric we therefore must have:  $A=B$ . Defining  $\gamma = |q|$  and substituting the particular solution and equation (A.5) into (A.3) yields equation (14) in the main text.

Finally, we can solve for b using the analytical approximation developed by Dixit (1991).

Dividing the VMC-condition (15) by the SPC-condition (16) yields:

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<sup>18</sup> This can easily be seen by plugging the particular solution into equation (13). The reason for this result is that the Bellman equation (12) is valid for  $x \in (-b, b)$  which is the region in which control is never

$$\frac{(e^{\gamma b} + e^{-\gamma b} - 2)}{\gamma b(e^{\gamma b} - e^{-\gamma b})} = \frac{1}{2} \left[ 1 - \frac{C\delta}{b^2} \right] \quad (\text{A.6})$$

Provided  $\gamma b$  is sufficiently small in a manner to be explained, the LHS can be approximated by a fourth-order Taylor expansion around  $\gamma b = 0$ :

$$\begin{aligned} & \frac{1 + \gamma b + \frac{1}{2}(\gamma b)^2 + \frac{1}{6}(\gamma b)^3 + \frac{1}{24}(\gamma b)^4 + (1 - \gamma b + \frac{1}{2}(\gamma b)^2 - \frac{1}{6}(\gamma b)^3 + \frac{1}{24}(\gamma b)^4) - 2}{\gamma b[1 + \gamma b + \frac{1}{2}(\gamma b)^2 + \frac{1}{6}(\gamma b)^3 + \frac{1}{24}(\gamma b)^4 - (1 - \gamma b + \frac{1}{2}(\gamma b)^2 - \frac{1}{6}(\gamma b)^3 + \frac{1}{24}(\gamma b)^4)]} = \\ & \frac{\gamma^2 b^2 + \frac{1}{12}\gamma^4 b^4}{\gamma b \left( 2\gamma b + \frac{1}{3}\gamma^3 b^3 \right)} = \frac{1}{2} \left( \frac{1 + \frac{1}{12}\gamma^2 b^2}{1 + \frac{1}{6}\gamma^2 b^2} \right) = \quad (\text{A.7}) \\ & \frac{1}{2} \left( 1 + \frac{1}{12}\gamma^2 b^2 \right) \left( 1 - \frac{1}{6}\gamma^2 b^2 \right) = \frac{1}{2} \left( 1 - \frac{1}{12}\gamma^2 b^2 \right) \end{aligned}$$

Equating the outcome of this approximation to the RHS of equation (A.6) and solving for  $b$  yields equation (17) in the main text.

Finally, we will examine under which conditions  $\gamma b$  will be sufficiently small. Take the following parameter values:  $\delta = 0.05$ ,  $\beta = 2$ ,  $\alpha = 1$ ,  $C = 0.01$  and  $\sigma = 0.1$ . Plugging these values into the expressions obtained for  $\gamma$  and  $b$  and subsequently computing the product

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exercised.

yields:  $\gamma b \approx 0.5$ . Since higher-order terms in the expansion of the LHS of (A.6) involve terms like  $(\gamma b)^5/120$  and smaller we can conclude that the approximation is quite robust.

## **Appendix B: Derivation of the probability of an interest rate decrease and the expected time period till next interest rate step**

Following Dixit (1993), let  $Q(x)$  denote that probability that  $x$  will first hit the lower barrier. Furthermore assume that  $x$  is regulated within the band  $(-a, b)$  where  $a, b > 0$ . For any  $x$  within this band it will hold that:

$$Q(x) = \frac{1}{2}Q(x - dx) + \frac{1}{2}Q(x + dx) \quad (\text{B.1})$$

Rewriting this equation and dividing by  $(dx)^2$  yields the following:

$$0 = \frac{[Q(x + dx) - Q(x)] - [Q(x) - Q(x - dx)]}{(dx)^2} \quad (\text{B.2})$$

Taking the limit of the RHS of this equation as  $dx \rightarrow 0$  we have:  $Q''(x) = 0$ . Therefore the general solution for  $Q(x)$  will be:

$$Q(x) = Fx + H \quad (\text{B.3})$$

where  $F$  and  $H$  are constants to be determined by examining  $Q(x)$  at the boundaries. This



yields:

$$\begin{aligned} Q(-a) = 1 & \Leftrightarrow -aF + H = 1 \\ Q(b) = 0 & \Leftrightarrow bF + H = 0 \end{aligned} \tag{B.4}$$

Solving for F and H we find:

$$Q(x) = \frac{b-x}{b+a} \tag{B.5}$$

Next, let  $T(x)$  denote the expected time period till the next interest rate step. For simplicity we assume that  $x$  is regulated within the symmetric band  $(-b, b)$ . For any  $x$  which is strictly in the interior of this band we have:

$$T(x) = dt + \frac{1}{2}T(x+dx) + \frac{1}{2}T(x-dx) \tag{B.6}$$

Rewriting this and dividing both sides by  $(dx)^2$  we have:

$$\frac{-2dt}{(dx)^2} = \frac{[T(x+dx) - T(x)] - [T(x) - T(x-dx)]}{(dx)^2} \tag{B.7}$$

From equation (11) it follows that  $(dx)^2 = \theta^2 \sigma^2 dt$ . Using this on the LHS of equation (B.7)

and subsequently taking the limit for  $dx \rightarrow 0$  on the RHS yields:

$$T''(x) = \frac{-2}{\theta^2 \sigma^2} \quad (\text{B.8})$$

Since the RHS of this equation is a constant we try a solution of the form:

$$T(x) = Lx^2 + Mx + N \quad (\text{B.9})$$

Using equation (B.8) it can be seen that  $L = -1/(\theta^2 \sigma^2)$ . Next, from the condition that

$T(-b)=T(b)=0$  we can establish:  $M=0$  and  $N = b^2/(\theta^2 \sigma^2)$ . Plugging these values into (B.9)

and using the expression obtained for  $b$  in equation (17) yields equation (19) in the main text.

## Appendix C: Derivation of the Long-Run Stationary Distribution for $x_t$

Consider the variable  $x_t$  which follows the Brownian motion described in equation (11)

and which is regulated within the band  $(-a,b)$  where  $a,b > 0$ . For any  $x_t \in (-a,b)$  let:

$$\begin{aligned} x_{t+dt} &= x_t + dx & \text{with prob } \frac{1}{2} \\ &x_t - dx & \text{with prob } \frac{1}{2} \end{aligned} \quad (\text{C.1})$$

From this, the stationary probability density function ( $\phi(x)$ ) must satisfy:

$$\phi(x) = \frac{1}{2}\phi(x-dx) + \frac{1}{2}\phi(x+dx) \quad (\text{C.2})$$

Rewriting this and dividing by  $(dx)^2$  yields:

$$0 = \frac{[\phi(x+dx) - \phi(x)] - [\phi(x) - \phi(x-dx)]}{(dx)^2} \quad (\text{C.3})$$

Taking the limit for  $dx \rightarrow 0$  on the RHS of (C.3), it follows that  $\phi''(x) = 0$ . Consequently, the general solution for  $\phi(x)$  will be:

$$\phi(x) = Fx + G \quad (\text{C.4})$$

where F and G are constants which can be determined by examining the behaviour of  $\phi(x)$  at the boundaries and the resetting point. First for  $x_t = -a + dx$  it will hold that:

$$\begin{aligned} x_{t+dt} &= -a + 2dx & \text{with prob } \frac{1}{2} \\ &= 0 & \text{with prob } \frac{1}{2} \end{aligned} \quad (\text{C.5})$$

From this we can conclude:

$$\begin{aligned}\phi(a) &= 0 \\ \phi(-a + 2dx) &= 2\phi(-a + dx)\end{aligned}\tag{C.6}$$

Furthermore, since  $\phi(-a+2dx)$  will satisfy equation (C.1), it can easily be shown that for  $n \geq 1$  and for  $-a < x < 0$ , it holds that:

$$\phi(-a + ndx) = n\phi(-a + dx)\tag{C.7}$$

Similarly, for the upper boundary  $b$  it can be shown that:

$$\begin{aligned}\phi(b) &= 0 \\ \phi(b - mdx) &= m\phi(b - dx)\end{aligned}\tag{C.8}$$

where the second equation holds for  $m \geq 1$  and  $0 < x < b$ . From equations (C.4), (C.7) and (C.8) we can see that  $\phi(x)$  will be linearly increasing in  $x$  for  $x \in [-a, 0)$  and linearly decreasing in  $x$  for  $x \in (0, b]$ . It remains to examine  $\phi(0)$  for which it holds that:

$$\phi(0) = \frac{1}{2}\phi(0 - dx) + \frac{1}{2}\phi(0 + dx) + \frac{1}{2}\phi(-a + dx) + \frac{1}{2}\phi(b - dx)\tag{C.9}$$

Rearranging and taking limits (see also Bertola and Caballero (1990)) we can write:

$$\lim_{dx \uparrow 0} \frac{\phi(0) - \phi(0 - dx)}{dx} + \lim_{dx \uparrow 0} \frac{\phi(b) - \phi(b - dx)}{dx} = \lim_{dx \downarrow 0} \frac{\phi(0 + dx) - \phi(0)}{dx} + \lim_{dx \downarrow 0} \frac{\phi(-a + dx) - \phi(-a)}{dx} \quad (\text{C.10})$$

From this equation it follows that while  $\phi(x)$  is continuous at  $x=0$  (this is ensured by (C.9)), it is not differentiable at this point since the RHS and the LHS derivatives at  $x=0$  have opposite signs.

Consequently, the Brownian motion process for  $x_t$  subject to barriers  $-a$  and  $b$  will give rise to a triangular steady state probability density function the support of which is determined by the control thresholds (see Figure 2).

Finally,  $\phi(0)$  can be determined by the requirement that  $\int_{-a}^b \phi(x) dx = 1$ . From Figure 2 it can be seen that this boils down to the condition that  $\frac{1}{2} (a+b) \phi(0) = 1$ . Using this, we obtain equation (20) in the main text.

## Appendix D: Proof of Propositions

### Proposition 2:

The sign of the partial derivatives of  $|i_\tau - i_0|$  with respect to  $C$ ,  $\beta$  and  $\sigma$  can be unambiguously inferred from equation (18). As for the parameter  $\alpha$  we can compute:

$$\frac{\partial |i_\tau - i_0|}{\partial \alpha} = \frac{(3\alpha - 2\beta)(6C\sigma^2)^{\frac{1}{4}}}{2\alpha^2(\beta - \alpha)^{\frac{3}{2}}} \begin{matrix} > \\ < \end{matrix} 0 \quad \text{if} \quad \begin{matrix} \frac{\beta}{\alpha} < \frac{3}{2} \\ \frac{\beta}{\alpha} > \frac{3}{2} \end{matrix} \quad (\text{D.1})$$

**Proposition 3:**

From equation (19) in the main text we can compute:

$$\begin{aligned} \frac{\partial T(x)}{\partial c} &= \frac{\sqrt{\frac{3}{2}}(\beta - \alpha)}{\sigma\sqrt{C}} > 0 \\ \frac{\partial T(x)}{\partial \beta} &= -\frac{\partial T(x)}{\partial \alpha} = \frac{\sigma\sqrt{6C} - 2(\beta - \alpha)x^2}{\sigma^2} \\ \frac{\partial T(x)}{\partial \sigma} &= -\frac{(\beta - \alpha)[\sigma\sqrt{6C} - 2(\beta - \alpha)x^2]}{\sigma^3} \end{aligned} \quad (\text{D.2})$$

From this it can be seen that for  $T(x)$  to be increasing in  $\beta$  and decreasing in  $\alpha$  and  $\sigma$  we need to have:

$$\sigma\sqrt{6C} > 2(\beta - \alpha)x^2 \Leftrightarrow |x| < \left( \frac{\sigma\sqrt{6C}}{2(\beta - \alpha)} \right)^{\frac{1}{2}} \Leftrightarrow |x| < \frac{b}{\sqrt{2}} \quad (\text{D.3})$$

**Proposition 4:**

From equation (20) in the main text we can compute (note that  $x \in (-a, b)$ ):

$$\frac{\partial Q(x)}{\partial b} = \frac{a+x}{(a+b)^2} > 0 \quad ; \quad \frac{\partial Q(x)}{\partial a} = \frac{x-b}{(a+b)^2} < 0 \quad (\text{D.4})$$

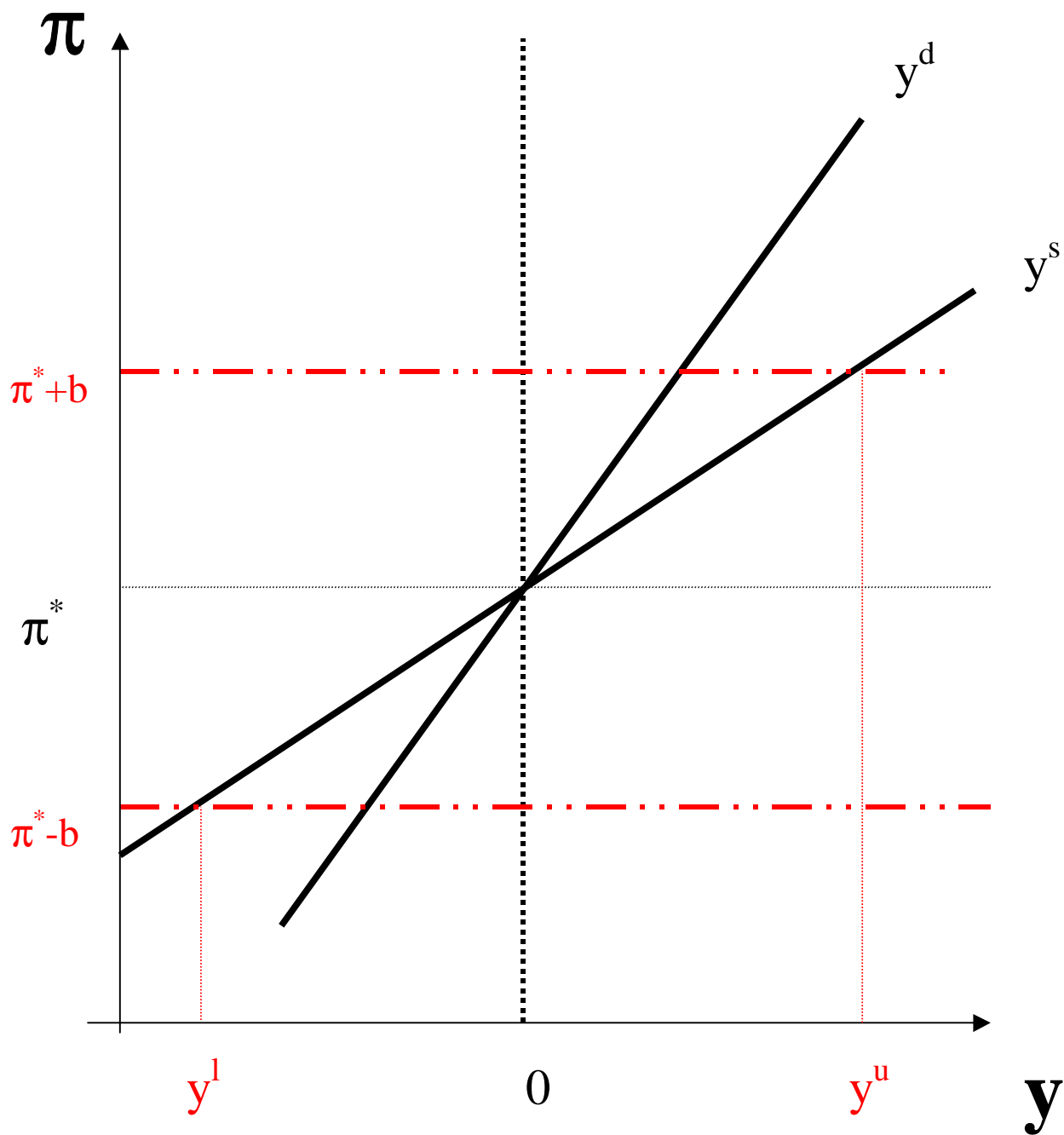
a and b (together with two constants of integration) are determined by the two Value Matching Conditions:  $L(-a)-L(0) = C_l$  ,  $L(b)-L(0) = C_h$  and the two Smooth Pasting Conditions:  $L'(-a) = L'(b) = 0$  . From these it can easily be shown that  $\partial b/\partial C_h > 0$  and that  $\partial a/\partial C_l > 0$  (see Dixit (1993)). Consequently we have:  $\partial Q(x)/\partial C_h > 0$  and  $\partial Q(x)/\partial C_l < 0$ .

**Proposition 5:**

To prove that  $\pi^e$  is increasing in  $C_h$  and decreasing in  $C_l$  it is sufficient to prove that this holds for  $E(x)$ . From equation (21) we have:

$$\frac{\partial E(x)}{\partial b} = \frac{1}{3} \quad ; \quad \frac{\partial E(x)}{\partial a} = -\frac{1}{3}$$

Using the results obtained in the proof of Proposition 4 then yields the proof of this proposition.

**Figure 1: The Optimal Band Width**



**Figure 2: Probability Density Function for the Inflation Gap**